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DETERMINATION OF THE DAMPING MOMENT IN YAWING FOR

TAPERED WINGS WITH PARTIAL-SPAN FLAPS

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SUMMARY

A method for determining the damping moment in yawing
for tapered wings with partial-span flaps is presented herein.
Charts are given for untwisted wings with taper ratios of
0.25, 0.50, and 1.00, with aspect ratios from 6 to 16, and
with center-span flaps extending from 25 to 100 percent of
the wing semispan. The results are also applicable to tip-
span flaps extending from 0 to 75 percent of the wing semi-
span. The calculated damping moment in yawing is compared
with experimental results for a rectangular wing with a flap
having a span 60 percent of the wing span.

INTRODUCTION

The calculation by Wieselsberger of the wing damping
moment for an untwisted elliptical wing in yawing is summa-
rized and extended in reference 1 to the case of an untwisted
rectangular wing. Reference 2 presents the results of cal-
culations for a wide range of taper ratio for untwisted wings
and also for the special angle-of-attack distribution that
results from the deflection of partial-span flaps of constant
chord ratio when the rest of the span is at zero angle of attack.

The results in reference 2 for the yawing derivative due to
induced drag, however, contain inaccuracies because of the
omission of an important term in the formula for the yawing
moment.   Also, as noted in reference 2, the results cannot
be applied by simple superposition to the case in which the
lift is contributed simultaneously by partial-span flaps and
by the plain portions of the wing.   This limitation follows
from the fact that the damping moment varies as the square of
the angle-of-attack distribution; hence, separate components
of the lift distribution have interactions that contribute to
the resultant value of the yawing derivative.

The present analysis gives the results of calculations
for the yawing derivative $\partial C_n / \partial \left(\frac{rb}{2V_s} \right)$ for untwisted tapered
wings with partial-span flaps of constant chord ratio at
various angles of attack.   The results are presented for the
same range of taper ratio as is considered in reference 2 and
for center-span flaps extending from 25 to 100 percent of the
wing semispan.   The results presented for the center-span
flaps may be applied to tip-span flaps extending from 0 to
75 percent of the wing semispan.   The computations in the
present paper do not include the part of the yawing moment
contributed by the changes in spanwise profile drag.   The
effect of this factor on the yawing derivative is discussed
in reference 2.

SYMBOLS

C_n yawing-moment coefficient due to spanwise induced-drag distribution

r angular velocity in yaw, radians per second

b wing span

V_s wind velocity along plane of symmetry of airplane

V local relative wind velocity at any section

y coordinate measured along lateral axis of airplane

Γ circulation around any section

c_l section lift coefficient

c wing chord at any section

α_i induced angle of attack at any section, radians

y_0 coordinate indicating fixed spanwise position

w normal component of velocity

N yawing moment due to spanwise induced-drag distribution $(C_n q_s S b)$

q_s dynamic pressure at plane of symmetry $\left(\frac{1}{2}\rho V_s^2\right)$

S wing area

ρ density

θ parameter defining spanwise position $\left(y = \frac{b}{2}\cos\theta;\right.$ when $\theta = 0$, $y = \frac{b}{2}$; when $\theta = \pi$, $\left. y = -\frac{b}{2}\right)$

θ_0 parameter defining fixed spanwise position

c_s wing chord at plane of symmetry of airplane

m_{0_s} slope of section lift curve at plane of symmetry of airplane, per radian

$A_1, \ldots A_n$ coefficients of Fourier series (see reference 3)

4

μ parameter $\left(\dfrac{c_s m_{o_s}}{4b}\right)$

A aspect ratio

C_{n_r} yawing derivative $\left[\partial C_n \Big/ \partial\left(\dfrac{rb}{2V_s}\right)\right]$

C_L over-all lift coefficient

$K_1, K_2,$ and K_3 proportionality constants used to express induced-yawing-moment derivative in terms of its component parts

λ taper ratio, that is, ratio of extended tip chord to chord at plane of symmetry

m_o slope of section lift curve, per radian

B arbitrary constant

ΔC_{L_f} increment of over-all lift coefficient due to deflection of flaps

b_f flap span

c_{d_o} section profile-drag coefficient

$\Delta c_{d_{of}}$ increment of section profile-drag coefficient due to deflection of flaps

ΔC_{n_r} increment of yawing derivative due to spanwise changes in profile drag

Subscripts and superscripts:

w refers to c_l- or c_{d_o}-distribution resulting from wing angle of attack

f refers to c_l- or c_{d_o}-distribution resulting from deflection of flaps

f_c center-span flaps

f_t tip-span flaps

METHOD AND ANALYSIS

The method used in the present analysis for calculating the stability derivatives is based on the assumptions outlined in reference 2. Because the angular motions considered in the present analysis are small ($rb/2V_s < 0.1$), the influence of the curvature of the wing wake is considered to be negligible for practical purposes. Powers of $rb/2V_s$ higher than unity are also neglected.

In yawing motion, the following relationships hold:

$$V = V_s \left(1 - \frac{ry}{V_s}\right)$$

where V is the local relative velocity at a section and y varies along the span from $\frac{b}{2}$ to $-\frac{b}{2}$

$$\Gamma = \frac{c_l c V_s}{2} \left(1 - \frac{ry}{V_s}\right) \tag{1}$$

$$a_{i_{y_o}} = \frac{w}{V} = \frac{1}{4\pi V} \int_{\frac{b}{2}}^{\frac{b}{2}} \frac{\frac{d\Gamma}{dy} dy}{y - y_o} \tag{2}$$

where $a_{i_{y_o}}$ is the induced angle of attack at any section y_o due to the trailing-vortex system.

$$N = -q_s \int_{\frac{b}{2}}^{-\frac{b}{2}} c_l c a_i \left(1 - \frac{2ry}{V_s}\right) y \, dy \tag{3}$$

where N is the yawing moment due to the spanwise changes in induced drag.

It is convenient to make the substitution

$$y = \frac{b}{2} \cos \theta$$

Then, by differentiating equation (1) with respect to the new variable θ, the downwash equation (2) becomes

$$\alpha_{i_{\theta_o}} = \frac{1}{4\pi b} \int_0^{\pi} \frac{\left[\frac{d(c_l c)}{d\theta} + c_l c \frac{rb}{2V_s} \sin \theta \right] d\theta}{\cos \theta - \cos \theta_o} \tag{4}$$

Equation (4) is equivalent to the downwash equation (8) given in reference 2.

If the Lotz method for determining the span load distribution (references 3 and 4) is followed, the circulation

$$\Gamma = \frac{c_l c V}{2} = \frac{c_s m_{o_s} V}{2} \sum_1^{\infty} A_n \sin n\theta \tag{5}$$

Substituting for $c_l c$, and $\frac{d(c_l c)}{d\theta}$ from equation (5) in equation (4) and integrating gives

$$\alpha_i = \frac{c_s m_{o_s}}{4b} \left[\sum_1^{\infty} n A_n \frac{\sin n\theta}{\sin \theta} - \frac{rb}{2V_s} \sum_1^{\infty} A_n \cos n\theta \right]$$

The yawing moment N from equation (3) now becomes

$$N = -q_s \frac{b}{16} \left(c_s m_{o_s} \right)^2 \int_0^{\pi} \left(-\sum_1^{\infty} A_n \sin n\theta \sum_1^{\infty} n A_n \frac{\sin n\theta}{\sin \theta} \right.$$

$$\left. + \frac{rb}{2V_s} \sum_1^{\infty} A_n \sin n\theta \sum_1^{\infty} A_n \cos n\theta \right) \left(1 - \frac{rb}{V_s} \cos \theta \right) \cos \theta \sin \theta \, d\theta \tag{6}$$

Integration of equation (6) and conversion to the nondimensional coefficient C_n results in

$$C_n = -\frac{\mu^2 \pi A}{4}\left\{ -\sum_1^\infty (2n + 1)A_n A_{n+1} \right.$$
$$\left. + \frac{rb}{2V_s}\left[\sum_1^\infty 2nA_n^2 - \frac{A_1^2}{2} + \sum_1^\infty (2n + 2)A_n A_{n+2}\right]\right\} \qquad (7)$$

where

$$\mu = \frac{c_s m_{0_s}}{4b}$$

It will be noted that the even-numbered A-coefficients in equation (7) are directly proportional to the asymmetric lift produced in yawing. These coefficients are therefore proportional to $rb/2V_s$ and their products or powers greater than unity can be neglected. It follows that, if the even-numbered A-coefficients have the values for unit $rb/2V_s$, equation (7) becomes

$$\frac{\partial C_n}{\partial \frac{rb}{2V_s}} = C_{n_r} = -\frac{\mu^2 \pi A}{4}\left[-\sum_1^\infty (2n + 1)A_n A_{n+1} + \sum_1^\infty 2nA_n^2 \right.$$
$$\left. - \frac{A_1^2}{2} + \sum_1^\infty (2n + 2)A_n A_{n+2}\right] \qquad (8)$$

In equation (8), each of the A-coefficients is a linear function of the angle-of-attack or c_l-distribution. For a given wing, therefore, similarly numbered A-coefficients due to any number of c_l-distributions may be superposed, and the

sum of these similarly numbered coefficients may be substituted in equation (8) to give the resultant value for C_{n_r} due to the combined lift distributions.

The subsequent analysis considers the case of the c_l-distribution caused by the combined distributions of the angle of attack and the deflection of partial-span flaps. The superscripts w and f refer to the c_l-distributions, due to the angle of attack and to the flap deflection, respectively. By combining these c_l-distributions, equation (8) becomes

$$C_{n_r} = -\frac{\mu^2 \pi A}{4}\left[-\sum_1^\infty (2n+1)\left(A_n^w + A_n^f\right)\left(A_{n+1}^w + A_{n+1}^f\right) + \sum_1^\infty 2n\left(A_n^w + A_n^f\right)^2\right.$$

$$\left. -\frac{\left(A_1^w + A_1^f\right)^2}{2} + \sum_1^\infty (2n+2)\left(A_n^w + A_n^f\right)\left(A_{n+2}^w + A_{n+2}^f\right)\right] \qquad (9)$$

Equation (9) may be expanded to the form

$$C_{n_r} = -\frac{\mu^2 \pi A}{4}\left\{\left[-\sum_1^\infty (2n+1)A_n^w A_{n+1}^w + \sum_1^\infty 2n\left(A_n^w\right)^2 - \frac{\left(A_1^w\right)^2}{2}\right.\right.$$

$$\left. + \sum_1^\infty (2n+2)A_n^w A_{n+2}^w\right] + \left[-\sum_1^\infty (2n+1)\left(A_n^w A_{n+1}^f + A_{n+1}^w A_n^f\right)\right.$$

$$\left. + \sum_1^\infty 4n A_n^w A_n^f - A_1^w A_1^f + \sum_1^\infty (2n+2)\left(A_n^w A_{n+2}^f + A_{n+2}^w A_n^f\right)\right]$$

$$\left. + \left[-\sum_1^\infty (2n+1)A_n^f A_{n+1}^f + \sum_1^\infty 2n\left(A_n^f\right)^2 - \frac{\left(A_1^f\right)^2}{2} + \sum_1^\infty (2n+2)A_n^f A_{n+2}^f\right]\right\}$$

$$(10)$$

The three groups of terms on the right-hand side of equation (10) are equal to the yawing derivative due to the w-distribution, to the interaction of the w- and f-distributions, and to the f-distribution. It should be noted that, in equation (10), the even-numbered A-coefficients have the values appropriate to unit $rb/2V_s$. In the equation, each A-coefficient is directly proportional to the over-all lift coefficient C_L that results from the particular c_l-distribution to which this A-coefficient refers. It follows that

$$C_{n_r} = \frac{C_{n_r}^{\,w}}{C_{L_w}^{\,2}}\,C_{L_w}^{\,2} + \frac{C_{n_r}^{\,wf}}{C_{L_w}\,\Delta C_{L_f}}\,C_{L_w}\,\Delta C_{L_f} + \frac{C_{n_r}^{\,f}}{\Delta C_{L_f}^{\,2}}\,\Delta C_{L_f}^{\,2} \tag{11}$$

where $C_{n_r}^{\,w}$, $C_{n_r}^{\,wf}$, and $C_{n_r}^{\,f}$ are obtained from equation (10), and the superscript wf denotes the interaction of the w- and f-distributions.

For a given wing, equation (11) may be written

$$C_{n_r} = K_1 C_{L_w}^{\,2} + K_2 C_{L_w}\,\Delta C_{L_f} + K_3\,\Delta C_{L_f}^{\,2} \tag{12}$$

where K_1, K_2, and K_3 are constants referring to the yawing derivative per unit C_L^2 for the angle-of-attack and flap-deflection distributions and for the interaction of these distributions.

RESULTS AND DISCUSSION

Theoretical Results

Calculations were made in the present analysis to deter-
mine the yawing derivative for the lift distribution that
results from a combined uniform and symmetrical distribution
of angle of attack and uniform deflection of partial-span
flaps of constant chord ratio.   The computations were made
by equation (10).   The A-coefficients for the angle-of-
attack and flap-deflection distributions were obtained by
the method given in references 2 to 4.   (See equation (9)
of reference 2.)   The computations assumed a value of 5.67
per radian for the slope of the section lift curve.   The
range of the investigation includes three taper ratios: 0.25,
0.50, and 1.00; three aspect ratios: 6, 10, and 16; and
flaps extending from the wing center to 25 to 100 percent
of the wing semispan.   The plan forms of the wings, which
have rounded tips, are shown in figure 1 of reference 2.
The results of the calculations are given in figures 1 to 3.
Figures 1 and 3 are similar to figures 13 and 12, respectively,
of reference 2.   The results presented in these figures in
reference 2 are in error because of the omission of the term
$-\frac{A_1{}^2}{2}$, which appears in equation (8) of the present paper.

The variation of the yawing-derivative factor $K_\downarrow$ with
aspect ratio is shown in figure 1 for taper ratios of 0.25,
0.50, and 1.00, for a uniform and symmetrical angle-of-attack

distribution, and for $\Delta C_{L_f} = 0.$ Figure 1 shows that the magnitude of K_1 decreases as the aspect ratio decreases and as the taper becomes sharper.

Figure 2 shows the variation with flap span of K_2, which gives the yawing derivative due to the interaction of the combined distributions of angle of attack and flap deflection. The actual computations for the taper ratios of 0.25, 0.50, and 1.00 were made for flap spans of 50 and 100 percent of the wing span. In order to obtain the correct fairing for the curves, similar results were computed for the elliptical wing with $A = 10$, for flap spans of 25, 50, 75, 87.5, and 100 percent of the wing span. The data of figure 2 indicate that the magnitude of K_2 decreases with aspect ratio and increases with flap span. For flap spans that are greater than approximately 50 percent of the wing span, the magnitude of K_2 increases as the wing becomes less tapered; the increase becomes greater as the flap span is increased. For flap spans of approximately 50 percent of the wing span, K_2 varies only slightly with wing taper. It is interesting to note in figure 2 that, for flaps of approximately 30 percent of the wing span, the curve for K_2 reverses sign; this condition indicates a destabilizing influence for this factor.

The variation of K_3 with flap span for uniform deflection of flaps of constant chord ratio and for $C_{L_W} = 0$

is shown in figure 3.   The actual computations and fairing
of the curves were obtained in the same manner as those for
figure 2.   The values for K_3 show variations with wing
taper, aspect ratio, and flap span similar to those for K_2
in figure 2.

The total value of the yawing derivative due to induced
drag for the combined distributions of angle of attack and
flap deflection is obtained from figures 1 to 3 and equa-
tion (12).

It should be noted from the derivation of equation (12)
that the values for C_{n_r} obtained from this equation and
also from figures 1 to 3 are equally applicable when the angle
of attack or the flap deflection is either positive or negative,
as long as proper account is taken of the signs.   Because
the analysis in deriving the formula for C_{n_r} shows that K_1,
K_2, and K_3 are negative for either a positive or a negative
angle of attack or flap deflection, the application of
equation (12) and figures 1 to 3 requires only a considera-
tion of the sign of C_{L_W} and ΔC_{L_f}.

Figures 1 to 3, as noted previously, are based on a
value of 5.67 per radian for the slope of the section lift
curve.   In order to apply the figures for other values of
m_0 - for example, $m_0 = 5.67B$, where B is an arbitrary
constant - each ordinate is multiplied by B^2 or each
curve in the figures can be shifted to an equivalent aspect

ratio BA, where A is the desired aspect ratio. The figures refer specifically to center-span flaps but the data may be applied to tip-span flaps of 0 to 75 percent of the wing span by the following relationship:

$$C_{n_r}^{w+f_t} = K_1 \left(C_{L_w} + C_{L_{f_{c+t}}}\right)^2 - K_2 \left(C_{L_w} + C_{L_{f_{c+t}}}\right)\Delta C_{L_{f_c}} + K_3\Delta C_{L_{f_c}}^2 \quad (13)$$

In the equation f_t refers to the tip flaps, f_c refers to center flaps for which the span is $\left(1.00 - \dfrac{b_{f_t}}{b}\right)b$, if $\dfrac{b_{f_t}}{b}$ is the ratio of the span of the tip flap to the wing span, and f_{t+c} indicates full-span flaps. The flap deflection corresponding to $C_{L_{f_{c+t}}}$ and $C_{L_{f_c}}$ is the same as that for the tip flaps. Equation (13), of course, can be used in conjunction with figures 1 to 3 only for the particular case of the combined c_l-distribution that results from a uniform deflection of flaps of constant chord ratio and from a uniform and symmetrical angle-of-attack distribution.

Comparison of Theoretical with Wind-Tunnel Results

The calculated results presented in figures 1 to 3 are compared in figure 4 with wind-tunnel results obtained from reference 5 for a rectangular wing with partial-span flaps of constant chord ratio. The wing has the following characteristics:

Tips .... Square
Aspect ratio, A 6
Taper ratio, λ... 1.00
b_f/b 0.60
ΔC_{L_f} 0.56
$c_{d_{o_w}}$, per unit wing span 0.024
$\Delta c_{d_{o_f}}$, per unit flap span 0.08

and

$$C_{L_w} = C_L - \Delta C_{L_f}$$

where C_L is the over-all lift coefficient. From figures 1 to 3, based on rounded tips, $K_1 = -0.0225$, $K_2 = -0.0219$, and $K_3 = -0.0125$; therefore, the yawing derivative due to induced drag

$$C_{n_r} = -0.0225 C_{L_w}^2 - 0.0219 C_{L_w} \Delta C_{L_f} - 0.0125 \Delta C_{L_f}^2 \qquad (14)$$

The yawing derivative due to profile drag from equation (12) of reference 2 is

$$\Delta C_{n_r} = -0.33(0.024) - 0.072(0.08) \qquad (15)$$

By combining equations (14) and (15), the yawing derivative for the wing in terms of the over-all lift coefficient becomes

$$C_{n_r} = -0.0225 C_L^2 + 0.01294 C_L - 0.01779$$

The comparison of the theoretical values with the wind-tunnel results is shown for a range of lift coefficient from 0 to 1.6. (See fig. 4.) It will be seen that the agreement generally is good except at high lift coefficients. The greater experimental damping moment at the high lift coefficients is probably due to the localized tip vortex caused by the square tips, which adds an increment of drag

increasing approximately as the square of the lift coeffi-
cient.

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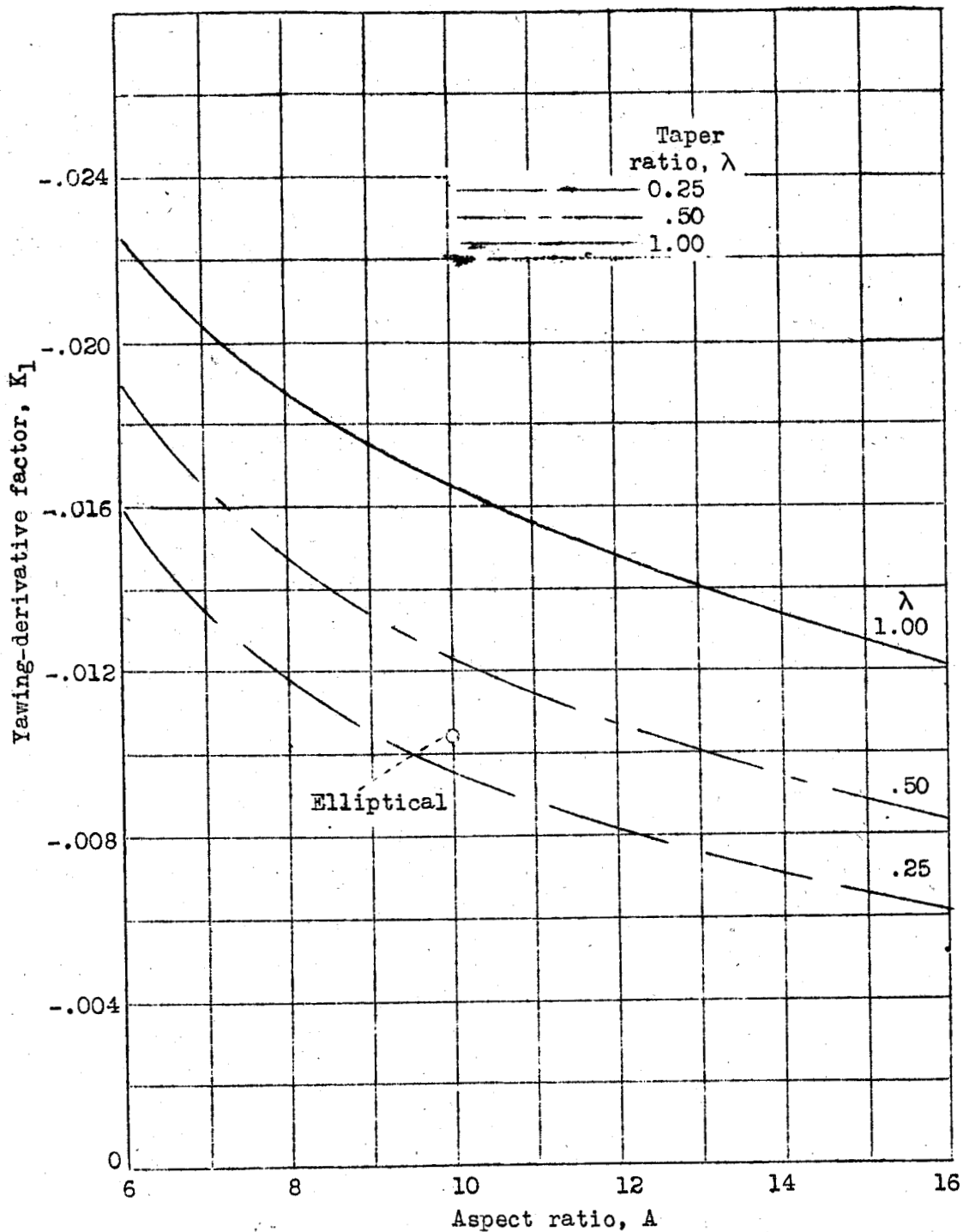
Figure 1.- Variation with aspect ratio of yawing-derivative factor due to induced drag for uniform and symmetrical angle-of-attack distribution. ΔC_{L_f}, 0. $C_{n_r} = K_1 \, C_{L_w}^2$.
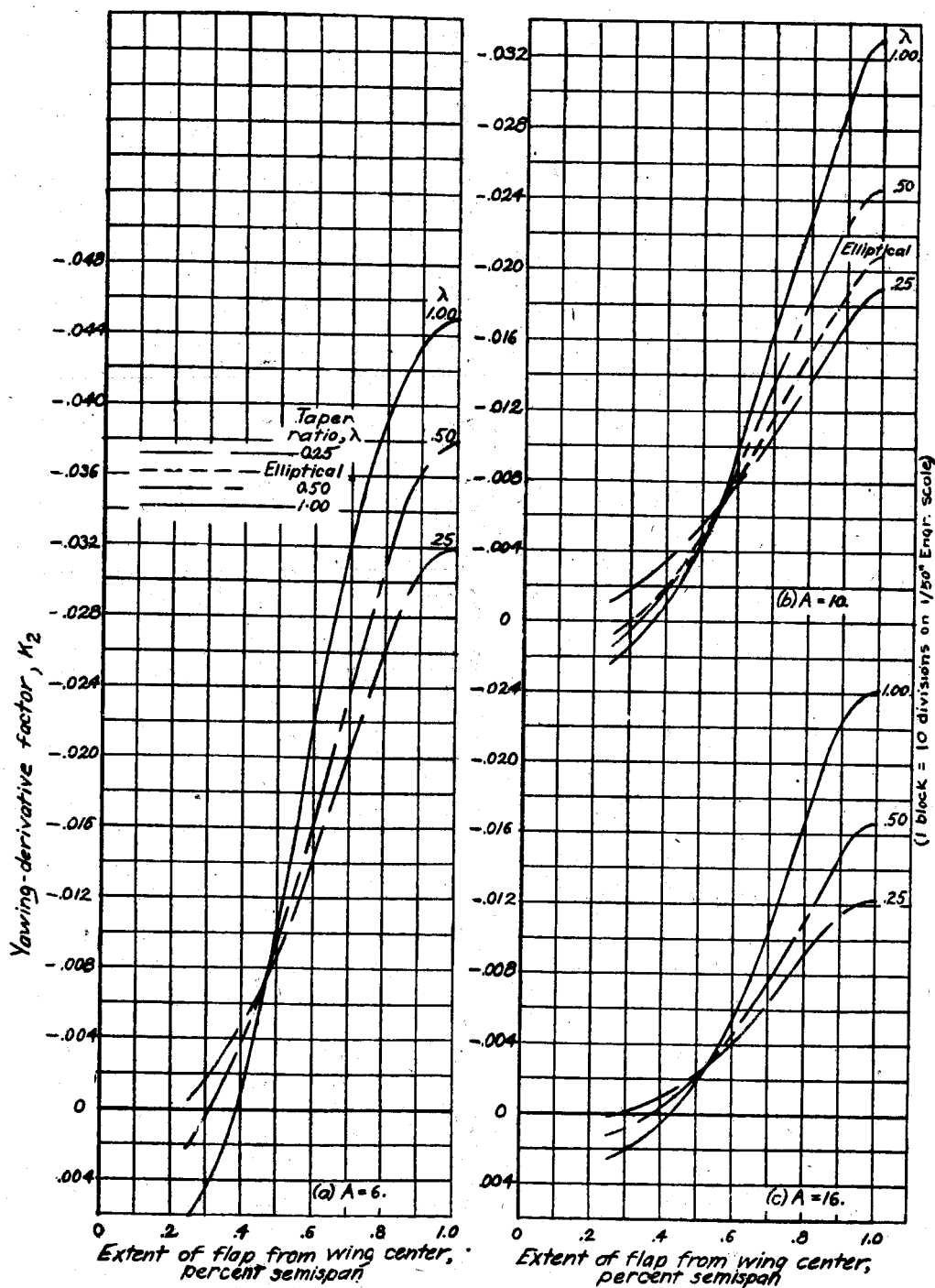
Figure 2.- Variation with flap span of yawing-derivative factor due to induced drag for interaction of uniform and symmetrical angle-of-attack distribution and uniform deflection of constant-chord-ratio flaps. $C_{n_r} = K_2 C_{L_w} \Delta C_{L_f}$.
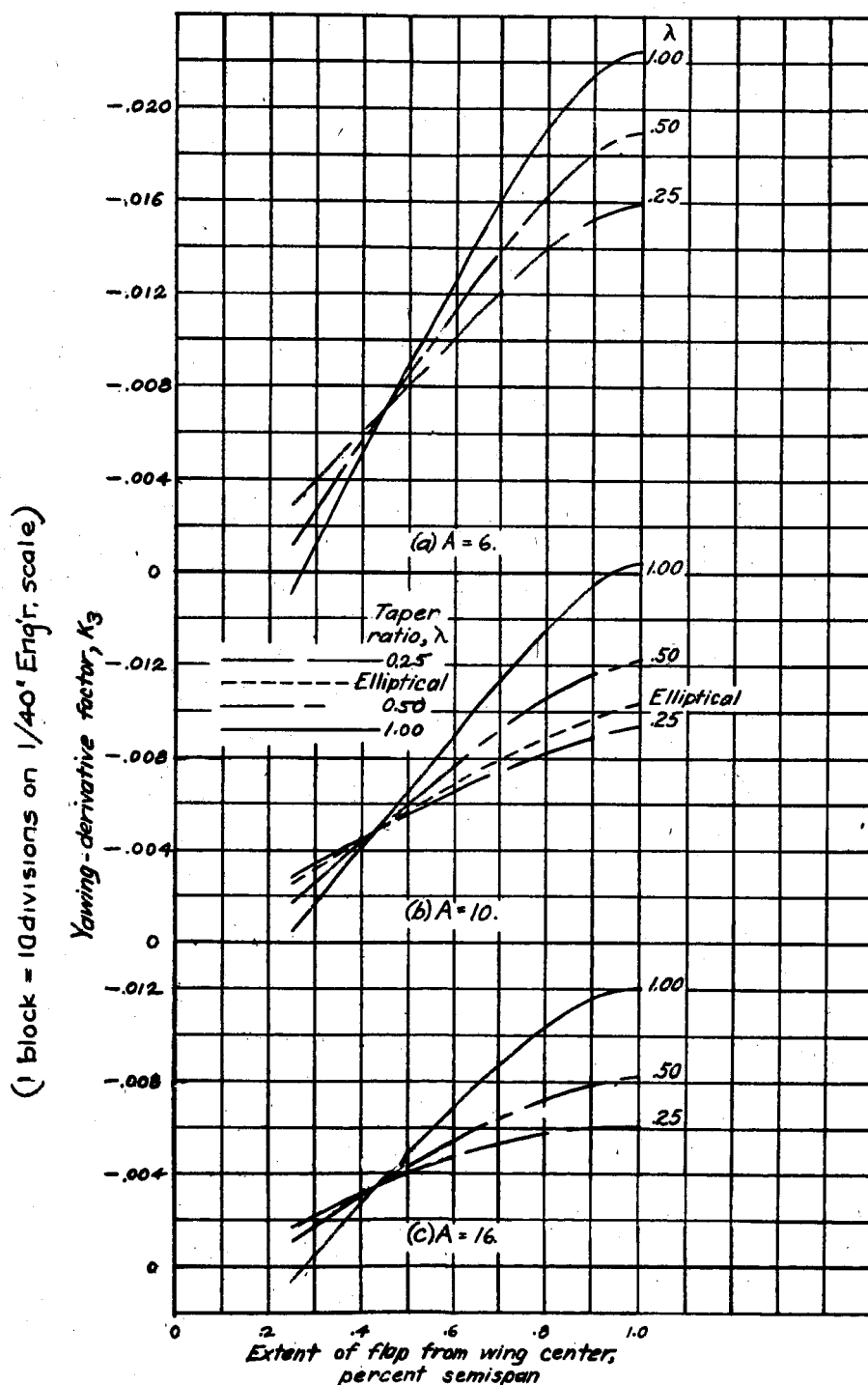
Figure 3.- Variation with flap span of yawing-derivative factor due to induced drag for uniform deflection of constant-chord-ratio flaps. c_{L_W}, 0. $C_{n_r} = K_3 \Delta C_{L_f}^2$.
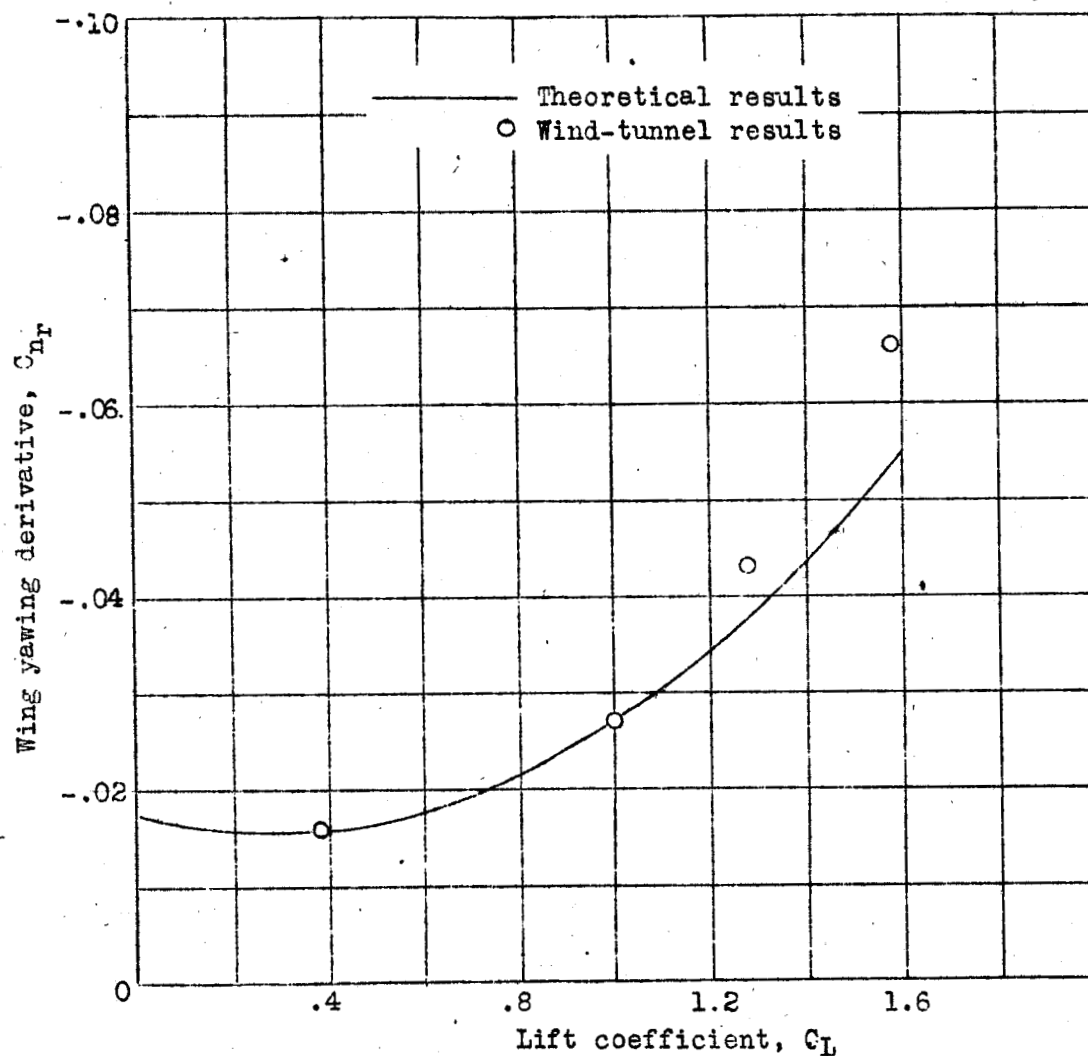
Figure 4.- Comparsion of theoretical and wind-tunnel results
for yawing derivate for partial-span flapped wing.
$A = 6$; $\lambda = 1.00$; $b_f/b = 0.60$; $\Delta C_{L_f} = 0.56$; and $C_{L_W} = C_L - 0.56$.